Exact solution of an electron kinetic equation for a strongly nonisothermal two-component plasma

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In the experiments on powerful femtosecond laser pulses interacting with matter, strongly nonisothermal plasmas can be generated [H. M. Milchberg, R. R. Freeman, S. C. Davey, and R. M. Moore, Phys. Rev. Lett. **61**, 2364 (1988)]. In the present paper it is shown that if electron and ion temperatures of plasmas satisfy the inequality $T_e \gg T_i$, the kinetic equation for the electron distribution function taking into account electron-ion collisions as well as electron-electron collisions can be solved exactly analytically. It is another example of the existence of an analytic solution of a kinetic equation in addition to the well-known case of the Lorentz model.

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I. INTRODUCTION

It is well known that when electron-electron (e-e) collisions in electron-ion plasmas are neglected, the kinetic equation for the electron distribution function in the slightly nonequilibrium case can be solved analytically [1]. It is also well known that if e-e collisions are taken into account along with electron-ion (e-i) collisions, then the kinetic equation can be solved only numerically [2] or approximately via expansion in Sonine-Laguerre polynomials [1].

In Silin's paper [3] it was demonstrated that for strongly nonisothermal plasmas where the electron temperature is much higher than the ion temperature ($T_e \gg T_i$), electron-electron interactions via plasma ion sound waves dominate in the e-e collision integral rather than conventional e-e binary collisions. Such an effect is negligible in e-i interactions because ion sound velocity is much greater than ion thermal velocity.

Silin and co-workers solved the electron kinetic equation for nonisothermal plasmas approximately, using the Sonine-Laguerre polynomial expansion [4,5]. Unfortunately, such an approach in the limiting case when e-e collisions dominate leads to an unphysical result for electrical conductivity [5].

The purpose of the present paper is to demonstrate that using Silin's result for the e-e collision integral in the case of a strongly nonisothermal plasma, one can solve the kinetic equation exactly analytically. This gives another example of an exactly solvable version of the kinetic equation in addition to the well-known Lorentz model, but in the rather more complicated case when light-particle–light-particle collisions are considered along with light-particle–heavy-particle collisions. It is now possible to examine separately, unlike the well-known Spitzer solution [2], the influence of e-e collisions on the electron free path.

The results of the present paper can be of importance in an analysis of the experimental results of ultrashort powerful laser pulses interacting with matter, particularly in reflectivity calculations [6,7].

II. COLLISION INTEGRALS FOR STRONGLY NONISOTHERMAL PLASMAS

The Boltzmann kinetic equation for the electron distribution function $f(p_j,t)$ in the external electric field E_j can be written in the following form:

$$eE_j\frac{\partial f}{\partial p_j}=I_{ei}+I_{ee} , \quad (1)$$

where I_{ei} and I_{ee} denote respective collision integrals and e is the electron charge.

Since the ion mass M is assumed to be much larger than the electron mass m, we can write [1]

$$\begin{aligned} I_{ei}&=n_i\frac{\partial}{\partial p_\alpha}B^{ei}_{\alpha\beta}\frac{\partial}{\partial p_\beta}f ,\\ B^{ei}_{\alpha\beta}&=B\Lambda\left[\delta_{\alpha\beta}-\frac{p_\alpha p_\beta}{p^2}\right] , \quad (2)\\ B&=\frac{2\pi e^4/Z^2m}{p} . \end{aligned}$$

where Z is the ion charge, n_i the ion number density, and Λ the conventional Coulomb logarithm [see Eq. (4)].

For e-e collision integrals we have the Balescu-Lenard expression

$$I_{ee}=n_e\frac{\partial}{\partial p_\alpha}\int d^3p' B^{ee}_{\alpha\beta}\left[\frac{\partial}{\partial p_\beta}-\frac{\partial}{\partial p'_\beta}\right]f(\mathbf{p},t)f(\mathbf{p}',t) ,$$

$$B^{ee}_{\alpha\beta}=2e^4\int_{-\infty}^{\infty}d\omega\int d^3k\,\delta(\omega-\mathbf{k}\cdot\mathbf{v})\times\delta(\omega-\mathbf{k}\cdot\mathbf{v}')\frac{k_\alpha k_\beta}{k^4|\epsilon(\omega,k)|^2} , \quad (3)$$

where $\epsilon(\omega,k)$ is the plasma dielectric function. It is well known [1] that for an isothermal plasma ($T_e=T_i$) the main contribution to $B^{ee}_{\alpha\beta}$ comes from the region $k>1/d$ that corresponds to conventional binary collisions where $d\sim d_e\sim d_i$ are Debye radii. Silin [4] drew attention to the fact that for a nonisothermal plasma ($d_e \gg d_i$), it is the long-wavelength region $1/d_e \ll k \ll 1/d_i$ that gives the dominant contribution to $B^{ee}_{\alpha\beta}$. This region corresponds to slightly attenuating ion sound waves, which are described by the roots of equation $\epsilon(\omega,k)=0$. Thus in a strongly nonisothermal plasma, electrons can exchange momentum not only in direct binary collisions, but also via emission and absorption of plasma ion sound waves. The analogous mechanism for an isothermal plasma has no significance. In that case only electron plasma waves with $k \ll 1/d_e$ can propagate without appreciable attenuation and therefore the number of electrons that can emit and absorb plasmons is exponentially small. Silin's formula for $B^{ee}_{\alpha\beta}$ [3] can be obtained from (3) by using the expression for the dielectric function of a strongly nonisothermal plasma [1]:

$$B^{ee}_{\alpha\beta}=\frac{n_\alpha n_\beta}{|\mathbf{p}\times\mathbf{p}'|}C\ ,\quad C=\frac{2(2\pi)^{1/2}e^4 Z T_e^{3/2} m^{3/2}}{T_i\Lambda_1}\ , \tag{4}$$

where n_α is the unit vector perpendicular to $\mathbf{p}$ and $\mathbf{p}'$, and $\Lambda_1=\ln(Z^2 M T_e^3/m T_i^3)$.

III. SOLUTION OF THE KINETIC EQUATION

For sufficiently small electric field E we can seek the solution of Eq. (1) in the conventional form

$$f=f_0+\delta f\ ,\quad \delta f \ll f_0 \tag{5}$$

where

$$f_0=n_e/(2\pi m_e T_e)^{3/2}\exp(-p^2/2m_e T_e)$$

and the deviation δf is expressed as

$$\delta f=\frac{p_j E_j}{m}g(p)f_0\ . \tag{6}$$

Substituting this expression into Eq. (1) and retaining only leading terms, one obtains the following equation for g:

$$\frac{e}{T_e}=2\frac{n_i B}{p^3}g(p)+\frac{2\pi^2}{p^3}C\int_0^\infty dp'p'f_0(p')[g(p)-g(p')]\ . \tag{7}$$

By direct substitution, one makes sure that the solution of Eq. (7) can be expressed in the form

$$g(p)=\frac{1}{\alpha}(p^3+\beta\gamma)\ , \tag{8}$$

where

$$\alpha=\frac{2T_e}{e}\left[n_i B+\pi^2 C\int_0^\infty dp\,pf_0(p)\right]\ ,\quad \beta=\frac{2T_e}{e}\pi^2 C\ , \tag{9}$$

and γ can be calculated from the equation

$$\gamma=\frac{1}{\alpha}\int_0^\infty dp\,f_0(p)(p^4+p\beta\gamma)\ . \tag{10}$$

Now it is not difficult to calculate electric conductivity via the conventional expression

$$\sigma=\frac{4\pi e}{3m^2}\int_0^\infty p^4 g(p)f_0(p)dp\ . \tag{11}$$

After calculating all integrals, one obtains

$$\sigma=\frac{8(T_e/\pi)^{3/2}}{Ze^2\sqrt{2m_e}\Lambda}\frac{\Lambda\Lambda_1+(\pi/64Z)T_e/T_i}{\Lambda\Lambda_1+\frac{1}{2}T_e/T_i}\ . \tag{12}$$

It can be observed from this result that if e-e collisions are neglected, expression (11) coincides with the well-known result for a Lorentz plasma. If e-e collisions are very strong (the collision integral tends to infinity), for instance, when $T_e \gg T_i$, the conductivity does not vanish. It is reduced with respect to the Lorentz value by a large, but nevertheless finite, factor $32Z/\pi$. If e-i collisions are neglected, but e-e ones are not, conductivity becomes infinite, as it should be from the common point of view, because e-e collisions do not change the total momentum of the electron subsystem. This result does not take place in [5], where expansion in Sonine-Laguerre polynomials was used to solve the kinetic equation.

It is also possible to solve the kinetic equation under conditions of the existence of space gradients of plasma parameters. For example, when $\nabla T_e \neq 0$, the kinetic equation is written as (temperature is measured in energy units)

$$\frac{f_0}{T_e^2}\left[\frac{p^2}{2m}-\frac{5T_e}{2}\right]\frac{p_j}{m}\nabla_j T_e=I_{ei}+I_{ee}\ , \tag{13}$$

the solution of which can be found as above:

$$f=f_0\left[1+\frac{p_j}{m}\nabla_j T_e g(p)\right]\ ,$$
$$g(p)=-\frac{1}{e\alpha}\left[\frac{p^5}{2mT_e}-\frac{5p^3}{2}\right]\ . \tag{14}$$

For instance, for the entire electron energy flux Q_j, one obtains

$$Q_j=\int\frac{p_j}{m}\frac{p^2}{2m}\delta f\,d^3p$$
$$=\nabla_j T_e\frac{5!\sqrt{2}T_e^{5/2}}{3\pi^{3/2}\sqrt{m}\,e^4 Z\Lambda}\frac{1}{1+(1/2\Lambda\Lambda_1)T_e/T_i}\ . \tag{15}$$

It can be seen from (11) that unlike electrical charge flux, energy flux remains finite even when e-i collisions are neglected.

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